The Fifth International Conference on History of Modern Mathematics

The "geometric orgies of Torino" Corrado Segre as the Founder of a School and as an Educator

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"*Anyone wishing to evaluate the work of Segre properly cannot disregard the contributions made by his School, the merits of which are in large part owing to him*" (Castelnuovo 1924, 358).

It is generally recognized that Segre was one of the most important architects of the "Risorgimento of geometry in Italy" (Coolidge 1904, 352)
Segre's university courses were a veritable forge for future researchers.

Under his guidance, in the period from 1891 to 1912, the Italian School of algebraic geometry was launched and, in just few years, Italy would assume a leading position (*führende Stellung*) on the international scene.

The flourishing of the Italian School of algebraic geometry
The "geometric orgies" (Castelnuovo 1893)


E. D'Ovidio


E. Bertini


G. Castelnuovo
(1887-1891)


C. Segre


G. Loria


M. Pieri
(1886-1900)

L. Viriglio
(1904)


G. Peano


G. Scorza
(1899-1900)


M. Bottasso
(1901)

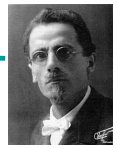

F. Enriques
(11.1893-
1.1894)


G. Fano
(1892)


F. Amodeo
(1890-91)

G. Giambelli
(1901)


A. Tanturri
(1899)


B. Levi
(1896)

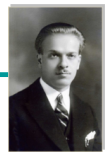

G. Gherardelli

G.


G. Chisholm
(1898-1899)


W.H. Young
(1898-1899)


F. Severi
(1900)


B. Segre
(1923)

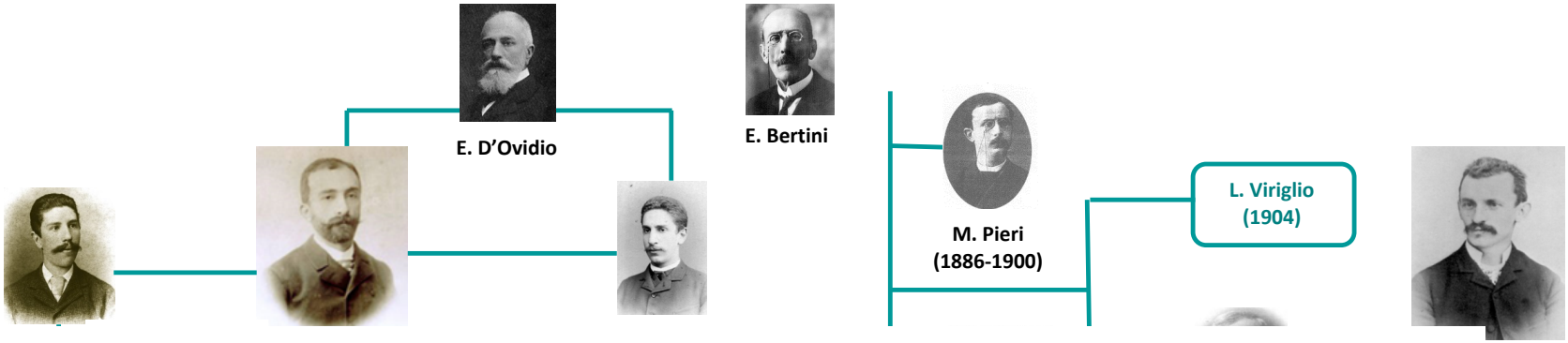
J. Coolidge
(1903-1904)

C. H. Sisam
(1908-1909)


E. Togliatti
(1912)

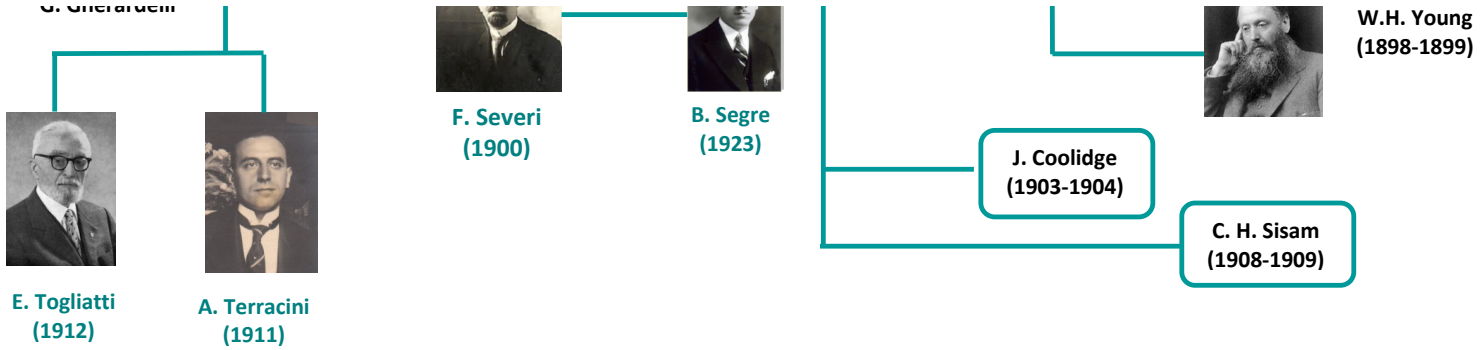

A. Terracini
(1911)

The flourishing of the Italian School of algebraic geometry
The "geometric orgies" (Castelnuovo 1893)



"It was indispensable that everything was treated and digested, that it became the blood of our blood, that we had it at our fingertips in order to be able to use it in the most advanced research … Fecundity!"

[Fano's ms, n.d., *Appunti vari*], fols 69r and 69v]



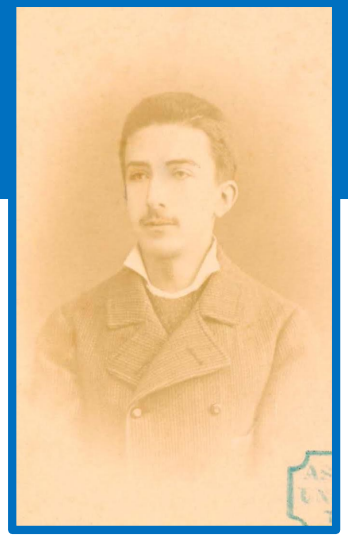
I will focus on:

- **the significant role of Segre's lessons in stimulating and fostering the research of his students,**
- **his approach to methodological questions connected with the teaching of mathematics in secondary schools**
- **his idea of "School".**

- documents of the Segre Archives
- forty handwritten notebooks of his university lectures:

http://www.corradosegre.unito.it/

Corrado Segre
Saluzzo 1863- Torino 1924



In **1883 Segre** received his degree at the University of Turin with a dissertation on hyperspatial projective geometry , a **"work of an experienced mathematician"** [Castelnuovo, 1924, 353]

- In 1888 he was called to the **chair of higher geometry at the University of Torino**, a chair that he would hold until his death;

- In **1898** the Accademia dei Lincei in Rome awarded *the Royal Prize for for Mathematics* jointly to Segre and Volterra;

- From **1887-88** to **1890-91** and from **1907-08** to **1920-21** he taught mathematics at the ***Scuola di Magistero*** (Teachers College) and in 1916 he was appointed director of this School;

- From **1909-10** to **1915-16** he was Dean of the Faculty of Science of the University of Turin;

- From **1904 to 1924** he was one of the editors of the ***Annali di Matematica pura ed applicata.***

The formation of the School: Interweaving of teaching and research

- **Segre's scientific contributions:**

He fostered the **onset or consolidation of the following lines of research**:

- Hyperspatial projective geometry ;
- Research in the foundations of hyperspatial projective geometry ;
- Birational algebraic geometry;
- Enumerative geometry;
- Projective differential geometry of hyperspaces;
- Projective geometry in the complex domain.

- **University teaching**

40 handwritten notebooks of his university lectures (1888-1924), the perfect example of a profound interaction between teaching and research.

[Giacardi 2001, 2003], [Conte, Giacardi 2016], [Conte 2016]

▶ Segre's notebooks are **an important documentation of his research work,** of which they sometimes constituted a "preliminary stage", sometimes a "reflection" (Terracini 1953, 261).

▶ They offer an extraordinary testimony of Segre's gifts as a teacher:

- In his courses he presented **the most recent studies** to his students;
- He suggested **topics for study**;
- He dealt with **problems still unsolved** with the aim of directing his students towards scientific research;
- He sometimes arrived at posing **new problems**.

Segre as *Maestro*

1887-1891 - Guido Castelnuovo (1865-1952) was in Turin and worked with Segre.
Their collaboration led **to the beginning of the Italian line of research on the theory of algebraic curves**.

To him I owe a good part of what I know; *in those long conversations, which we had two or three times a day during my stay of four years in Torino,* ***I learned more than in my university courses.*** (Castelnuovo to O. Michelli Segre, 25 May 1924)

C SEGRE 1894, *Introduzione alla geometria sopra un ente algebrico semplicemente infinito*, Annali di Matematica pura ed applicata, s. 2, 22, 41-142; *Opere*, 1, 198-304):
«the roots» of Italian algebraic geometry

Correspondence Segre-Castelnuovo: **255 letters from 1885 to 1905** (Acc. Lincei, Archivio Castelnuovo)

1890-1891 Segres's university course - ***Introduzione alla geometria sugli enti algebrici semplicemente infiniti***

(http://www.corradosegre.unito.it/Quaderni/Quad3/1_3.php)

"In the 1890-91 academic year Segre repeated with D'Ovidio in Torino the excellent co-teaching experiment made by Brioschi, Casorati and Cremona in 1869 in Milan . While D'Ovidio gave a course of lectures on Functions of complex variables and Abelian integrals, *Segre taught* Geometry on a simply infinite algebraic variety *from three points of view,* ***hyperspatial, algebraic and functional."*** (Amodeo 1945, 245).

Federico Amodeo
(1859-1946)

Problem submitted by Segre : "*Define the space S_r not by means of coordinates, but rather by a series of properties from which the representation with coordinates can be deduced as a consequence.*"

Gino Fano
(1871–1952)

(Amodeo 1891; Fano 1892; Fano 1895)

July 1890 - Eugenio Bertini came to Turin to learn from Segre "how to treat the curves and ruled surfaces of hyperspace with our works and methods" (C. Segre to G. Castelnuovo, Torino, 28 July 1890).

Eugenio Bertini
(1846-1933)

Twofold importance of this course:

♦ **scientific significance of Segre's approach** to the study of the geometry on curves,

♦ Segre's role as the leader of the School began.

He was fully **aware of :**

- **the significance** of the new line of research;
- the **presence of a group of researchers** who shared the new approach;
- the **importance of spreading their ideas and methods.**

"We really must think about writing treatises, lithographing lessons, spreading our ideas widely" (C. Segre to G. Castelnuovo, Torino, 6 July 1890)

Segre was aware he was creating a School.

Segre as *Maestro*
Towards an international dimension

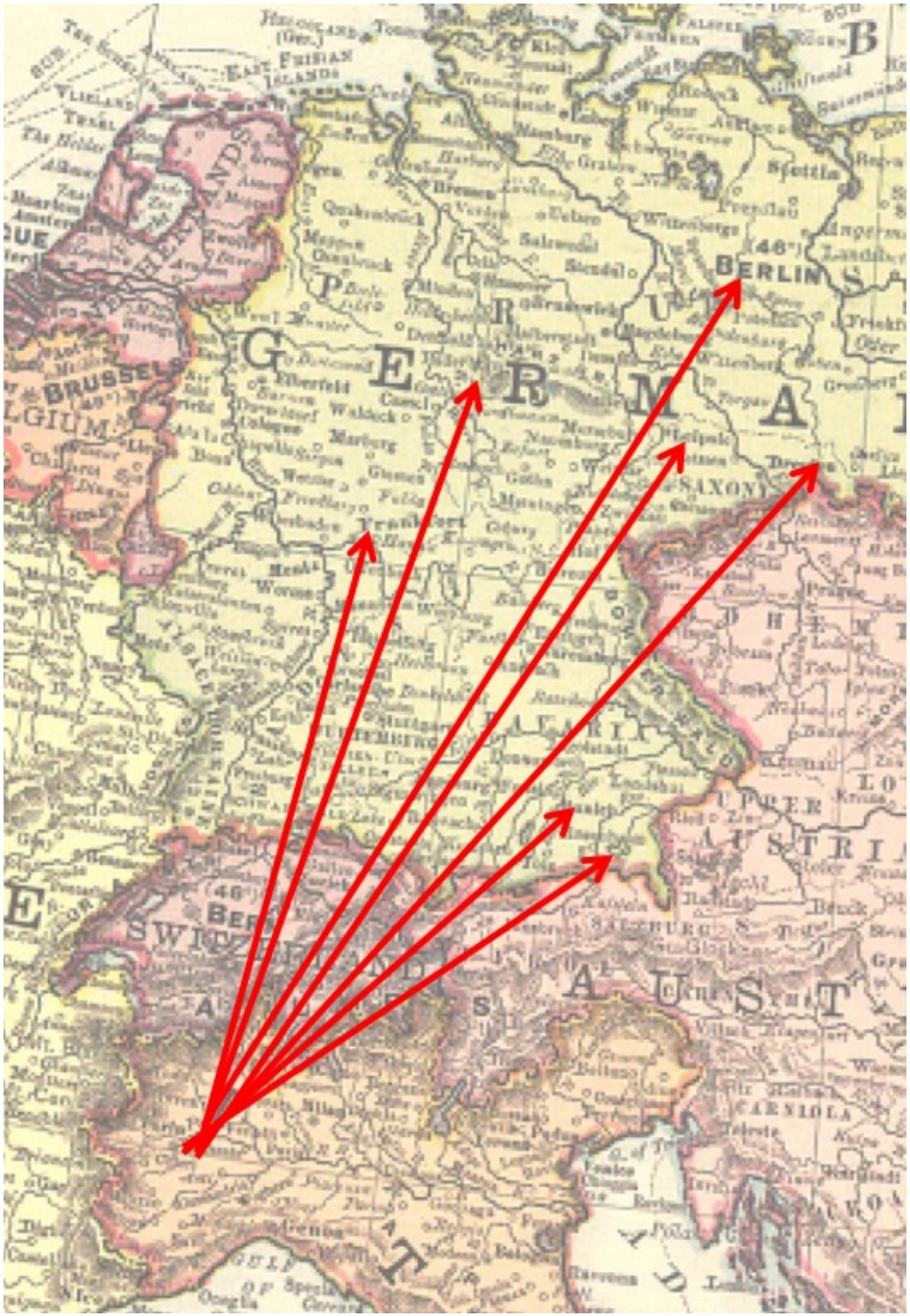
1890 On Segre's suggestion Fano undertook the Italian translation of Klein's Erlangen Program:

G. FANO 1890, F. KLEIN, *Considerazioni comparative intorno a ricerche geometriche recenti* (traduzione), Annali di matematica pura ed applicata, s. 2, 17, 307-343.

*"**This work is not, in my opinion, well enough known to the** **young Italian geometers**; and it is especially for them that I wished it to be reprinted. So many general and ingenious ideas are to be found in these pages, such as **the substantial** **identity among various mathematical disciplines** (and in particular between analytical and geometrical disciplines!) which can be seen to overlap one another when account is taken of the* transformations groups *that form the basis of them."*
(Segre, [Nota] 1890, 307-308)
See also (Segre to Klein, 19 November 1889)

Felix Klein
(1849-1925)

(Luciano, Roero 2012)



Summer 1891 - Segre's travel to Germany

He visited Göttingen, Frankfurt, Nuremberg, Berlin, Dresden, Leipzig and Munich,

He met **L. Kronecker, K. Weierstrass, M. Nöther, T. Reye, R. Sturm, K. Rhon, M. Cantor and F. Klein**:

"No one who hasn't been here can imagine the kind of man Klein is and the kind of organisation he was able, … to impose on the mathematical studies of this University; ***it is something that has made an enormous impression on me"***

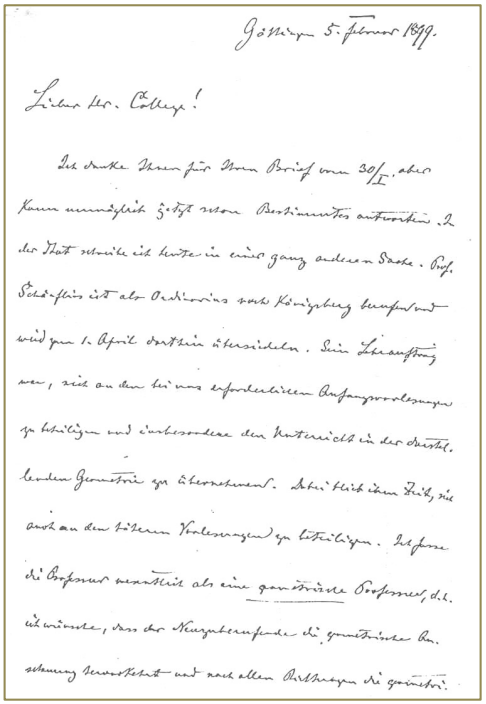
(Segre to Castelnuovo, Göttingen, 30 June 1891).

1892 - Fano graduated with a dissertation on hyperspatial geometry (Fano 1894, Segre's report 1892-93): influence of both Segre and Castelnuovo.

1893-94 - On Segre's recommendation **Fano spent a period of post-graduate study in Göttingen with Klein.**
He attended two seminars, in the WS 1893-94, and in SS 1894.
Fano's talks at the Mathematische Gesellshaft: **dissemination of the results of the Italian School of geometry.** (*Fondo Fano*, Scritti 4)
http://www.uni-math.gwdg.de/aufzeichnungen/klein-scans/klein/

1899 - Klein offered Fano a chair of geometry in Göttingen:

"I conceive the chair essentially as a chair of geometry, *that is, I wish the one who holds it* ***to exalt geometric intuition and develop geometric studies in all directions*** *… I have reached the conclusion that* you *are precisely the man for us!"*
(Klein to Fano, 5 February 1899, *Fondo Fano*, Lettere 9)



Fano preferred a chair in an Italian university: in 1901 he obtained the professorship in Turin.

Federigo Enriques
(1871-1946)

November 1893 – January 1894
Federigo Enriques joined Segre's group.
1894 - Castelnuovo and Enriques submitted the problem of the resolution of the singularities of algebraic surfaces to Segre (Letters exchanged among Segre, Castelnuovo, Enriques) (Bottazzini, Conte, Gario 1996)

1894-95 course - *Teoria delle singolarità delle curve e superficie algebriche* (BMP - *Fondo Segre* - Quaderni. 6)

As to my course, as long as we are dealing with easy ordinary or not ordinary singularities, I will put curves and surfaces together. ***But when we go to higher singularities I will have to make a distinction: first curves and then surfaces.*** *And to tell you confidentially,* ***I am not sure that there will be enough time for the last of these!*** (Segre to Castelnuovo, 24 December 1894)

Beppo Levi
(1875-1961)

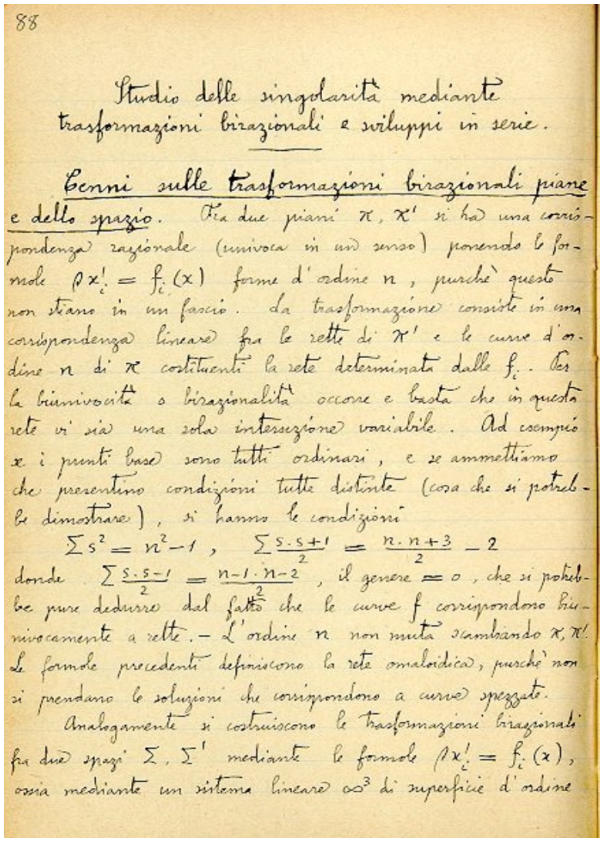
Beppo Levi attended this course and graduated in 1896. Segre involved him in the study of the singularities of algebraic surfaces.

(Gario 2016)

1896-97 course - *Lezioni sulle singolarità delle curve e superficie algebriche* (BMP - *Fondo Segre* - Quaderni. 8)

Segre presents the theory of singularities of algebraic surfaces, following the publication in **1897** of his important paper

C. SEGRE, *Sulla scomposizione dei punti singolari delle superficie algebriche*, Annali Mat. pura ed applicata, 2, 25, 1897 pp. 2-54; *Opere*, 1, 327-379)



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Studio delle singolarità mediante trasformazioni birazionali e sviluppi in serie.

Cenni sulle trasformazioni birazionali piane e dello spazio. Tra due piani π, π' si ha una corrispondenza razionale (univoca in un senso) ponendo le formole $\rho x'_i = f_i(x)$ forme d'ordine n, purchè questo non stiano in un fascio. La trasformazione consiste in una corrispondenza lineare fra le rette di π' e le curve d'ordine n di π costituenti la rete determinata dalle f_i. Per la biunivocità o birazionalità occorre e basta che in questa rete vi sia una sola intersezione variabile. Ad esempio se i punti base sono tutti ordinari, e se ammettiamo che presentino condizioni tutte distinte (cosa che si potrebbe dimostrare), si hanno le condizioni

$$\sum s^2 = n^2 - 1, \quad \sum \frac{s(s+1)}{2} = \frac{n(n+3)}{2} - 2$$

donde $\sum \frac{s(s-1)}{2} = \frac{(n-1)(n-2)}{2}$, il genere $= 0$, che si potrebbe pure dedurre dal fatto che le curve f corrispondono biunivocamente a rette. – L'ordine n non muta scambiando x, x'. Le formole precedenti definiscono la rete omaloidica, purchè non si prendano le soluzioni che corrispondono a curve spezzate.

Analogamente si costruiscono le trasformazioni birazionali fra due spazi Σ, Σ' mediante le formole $\rho x'_i = f_i(x)$, ossia mediante un sistema lineare ∞^3 di superficie d'ordine

In **1897-98 Beppo Levi** proves the resolution theorem of the singularities of surfaces.

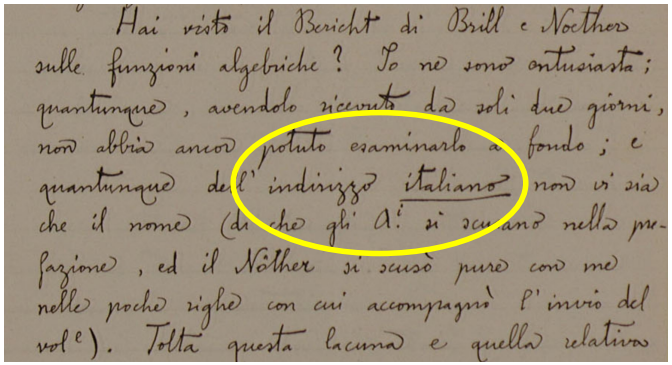
1935 revision by Oscar Zariski

Zariski 1971, 13:
Segre laid down the foundations of a geometric theory of singularities of algebraic surfaces.

In the mid-1890s Segre and his team – Fano, Castelnuovo and Enriques – already enjoyed the high **esteem of the international community**:

- they were invited to collaborate on the ***Encyklopädie der mathematischen Wissenschaften*** (5 articles)
- **1894** Brill and Nöther, in the preface to their ***Bericht***, *die Entwicklung der Theorie der algebraischen Functionen in älterer und neuer Zeit* made reference to the new line of research (***neuer Weise***) of the Italians. (Brill, Nöther 1894, IV)

(C. Segre to G. Castelnuovo, Torino, 28 November 1894)


Hai visto il Bericht di Brill e Noether sulle funzioni algebriche? Io ne sono entusiasta; quantunque, avendolo ricevuto da soli due giorni, non abbia ancor potuto esaminarlo a fondo; e quantunque dell'indirizzo italiano non vi sia che il nome (di che gli A.i si scusano nella prefazione, ed il Nöther si scusò pure con me nelle poche righe con cui accompagnò l'invio del vol.e). Tolta questa lacuna e quella relativa

- **1897 -** Segre was invited to be vice-president of the session of geometry at the International Congress of Mathematician in Zurich
- **1897 -** in the Introduction of the first volume of the treatise Picard & Simart *Théorie des fonctions algébriques de deux variables indépendantes* (1897, 1906) the authors cite the research of Castelnuovo and Enriques ***"qui ont renouvelé toute une partie de la Théorie des surfaces"*** (I, p. VI).

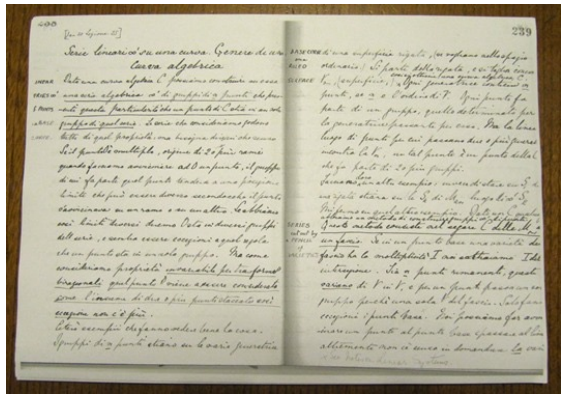
- **Mathematicians from abroad came to Turin to attend Segre's lessons, especially from England and North America**

1898-99 course - *Lezioni sulle curve algebriche dei vari spazî*

(BMP - *Fondo Segre* - Quaderni. 12)

William H. Young
(1863-1942)

Grace Chisholm
(1868-1944)



Letters and 5 Youngs' handwritten notebooks related to Segre's lectures and Grace's *Diario (Archives of the Univ. of Liverpool)* testify to the Youngs' relationship with Segre.

"to live a lifetime of mathematics"

"If you and your husband ***would like for me to come one day and speak with you about a geometric subject that interests you,*** *I pray you to write me and indicate the topics"* (Segre to G. Chisholm, 11 March [1899])

The Notebooks by Grace and William are particularly interesting:

- **the treatment is much more extensive** than in Segre's notebooks; it is possible to see the changes made by Segre during the lesson with respect to what he had planned to do;
- **applications of the theory and exercises** proposed by Segre are included;
- mention is made of **new research** he submitted to his students, and of the **extra-curricular lectures** (including evening lectures);
- the most recent works to be consulted are indicated,
- and advice for study or research also appear.

In 1899 Segre presented two paper of the Youngs for publication

1911 – At Segre's suggestion Luisa Viriglio undertook the Italian translation of *A First Book of Geometry* (1905) written by Grace and William Young.

Experimental and dynamic teaching of geometry.

1899-1900 course - *Lezioni di Geometria numerativa*

(BMP - *Fondo Segre* - Quaderni. 13)

"At the University this year I will treat … enumerative geometry. There ***are from 15 to 20 students, including three ladies. There are also five doctors.*** *I hope that at year's end I find myself content with the profit of these listeners"*
(Segre to G. and W. Young, 30 November 1899)

A. Tanturri
(1877-1924)

Segre's interest in enumerative geometry (Segre 1897-1898, Segre 1898, Segre 1900) inspired important research of **Tanturri, Severi** and **Giambelli** who graduated under Segre's advisement in **1899**, **1900** and **1901** respectively.

F. Severi
(1879-1961)

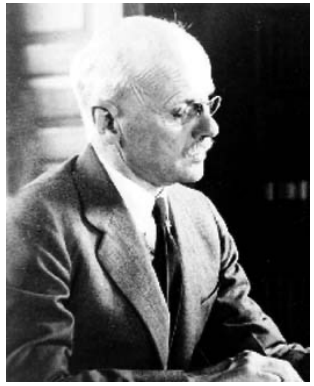
Francesco Severi discussed a thesis on singularities of curves in a hyperspace.

"Segre was quick to recognise the exceptional talent of his young pupil who, ***under his master's direction, soon began to produce original work of high quality;*** *in fact Severi's doctoral thesis, written in 1900, still retains interest."* (Roth 1963, 282)

1900-1903 - Segre presented **12** papers by his three students (Tanturri, Severi and Giambelli) for publication

ICM 1904, Heidelberg – Segre's plenary lecture

1903-1904 course - *Applicazioni degli integrali Abeliani alla Geometria* (BMP - *Fondo Segre* - Quaderni. 17)
This course was attended by **Julian Coolidge (1873-1954).**
His stay in Turin influenced his early scientific work on topics of **complex projective geometry.**


J. Coolidge
(1873-1954)

"There was no limit to the amount of ***care and patience*** *which he would bestow on one of his pupils"* (Coolidge 1927, 357).

"Every student of geometry in complex domain will find that he is forced to refer continually to the work of two admirable contemporary geometers, ***Professor Corrado Segre of Turin****, and Professor Eduard Study of Bonn. The names of both appear incessantly throughout this book;* ***the author had the rare privilege to be the pupil of each of these masters...*** *and he begs to offer the present work as a small token of* ***admiration and esteem.****"*
(Coolidge, *The geometry of the complex domain,* 1924, 7) (Brigaglia 2016)

Corrado Segre in 1899

The work carried out within Segre's group in this period actually was a *"**collective researches** – Segre-Castelnuovo: 1890-91 in Turin – Castelnuovo Enr[iques] (1896-900) afterwards Severi for surfaces (irraz. 1904-05). **Energies of investigators are summed. Their discoveries follow each other rapidly**"* (original English text by Fano, [Fano n. d.]., fol. 84v)

A **collective project** dates back to the end of the 19th century:
a treatise of higher geometry
Vorlesungen über höhere algebraische Geometrie.

For several years the announcement of this book appeared in the Teubner Catalogue, but the treatise never came to light. (Gario 2016)

Turin at the end of the 19th century

1907-08 course - ***Capitoli vari di Geometria della retta***

(*BMP - Fondo Segre - Quaderni. 21*)

C. SEGRE, *Su una classe di superficie degl'iperspazi legate colle equazioni lineari alle derivate parziali di 2° ordine,* Atti della R. Accademia delle Scienze di Torino, 17, 1906-07, 1047-1079; *Opere* 2, 20-49: **this work paved the way in Italy for the flourishing of projective differential geometry of hyperspace.**

Alessandro Terracini
(1889 – 1968)

1911, 1912 - Alessandro Terracini and Eugenio Togliatti (1890-1977) graduated, discussing theses directed by Segre.

1908-09 – Charles H. Sisam (Cornell University) followed Segre's lectures

"Beginning with a very significant memoir in 1907 ***Segre … was a leader in studying the projective differential geometry of hyperspace****. Segre gave analytic proofs regularly, but was also an outstanding exponent of the synthetic method, making differential properties even in hyperspace appear intuitive."* [Lane 1932, 288]

References to Segre's lesson notebooks in scientific literature

- **F. Enriques** used Segre's notebooks to write his *Conferenze di Geometria* (1894-95): *"I still have the notes from the lessons of Segre that Fano loaned me; please ask him to forgive me and ask if I may keep them until I have finished preparing the final lithographs of my lectures"* (Enriques to Castelnuovo, Ancona 20 July 1893);

- **R. Bonola** *La geometria non euclidea* (1906, Introduzione, XVII);

- In the preface to his treatise *Introduzione alla geometria proiettiva degli iperspazi* (1907) **E. Bertini** wrote that he had consulted "*the extensive manuscript summaries that Segre himself compiled annually for his courses*"(V);

- **F. Severi** used them in his *Trattato di Geometria algebrica* (1926, chapter on geometry of algebraic curves);

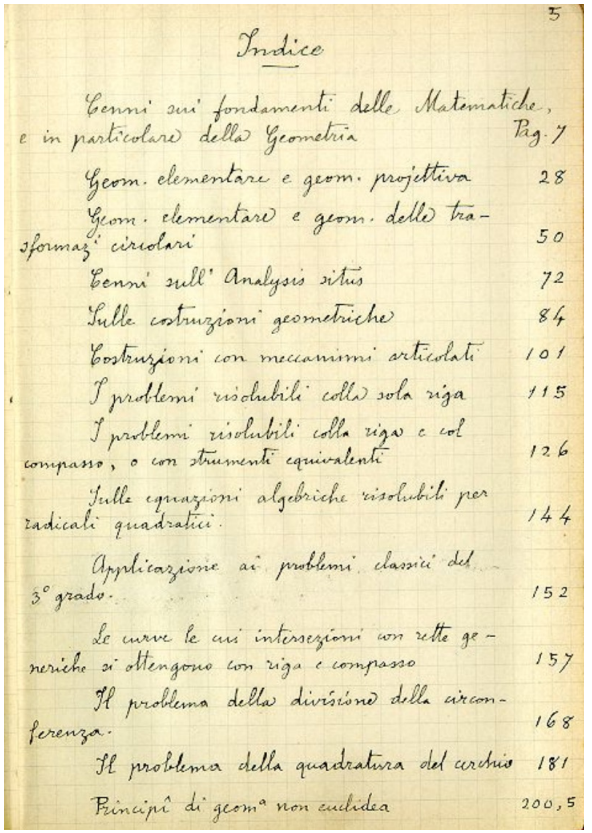
- **F. Enriques** and **O. Chisini** cite Segre's notebooks in their *Lezioni sulla teoria geometrica delle equazioni e delle funzioni algebriche* (1915-1934, II, 541 and III, 154).

The courses at the *Scuola di Magistero*
1887/88 – 1890/91 and 1907/08 – 1920/21

Vedute superiori sulla geometria elementare (1916-17)

(*BMP - Fondo Segre - Quaderni. 30*) **Scientific aspects**

non-Euclidean geometry, foundations of geometry, elementary geometry and projective geometry, geometrical constructions, linkages, problems that can be solved with straightedge and compass, algebraic equations that can be solved by extracting square roots, the cyclotomy problem; the problem of squaring the circle.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Indice

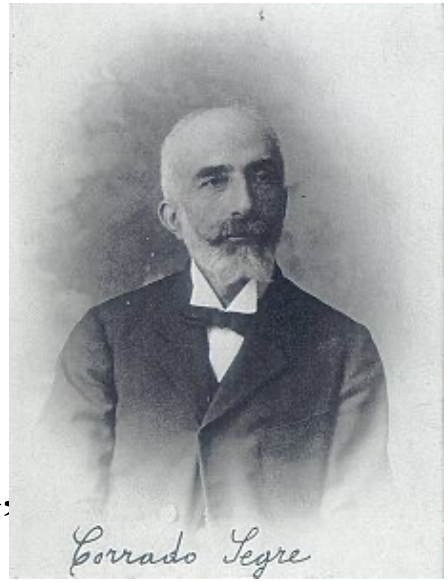
→ **Klein**

[Notes concerning the lectures given at the Scuola di Magistero]

(*BMP - Fondo Segre - Quaderni. 40*) **Didactic aspects**

Mathematics and experience; Mathematics and applications; Mathematics as a logical science; The aim of mathematics teaching in secondary schools; Intuition and the postulates; Rigour; methods; exercises.

Two ways of addressing mathematics



- Considering mathematics in relation to applications (**Klein**):

- gathering information derived from experience,
- putting the data obtained into mathematical form and proceeding to a purely mathematical treatment of the problem,
- translating the mathematical results into the form most suitable for the applications.

- Considering mathematics from an exclusively logical point of view (**Peano, Hilbert**)

Segre preferred **the first:**

"*Let us say immediately that this second line is of* ***great importance, philosophically as well****. It has made it quite clear what pure mathematics is; and has contributed to making various parts of mathematics more rigorous. But, by* ***detaching itself from reality, there is a risk of ending up with theoretical constructions, which, even while logical, are too unnatural, and cannot be of lasting scientific importance****."* (p. 13-14)

Segre's tenets - Influence of Klein

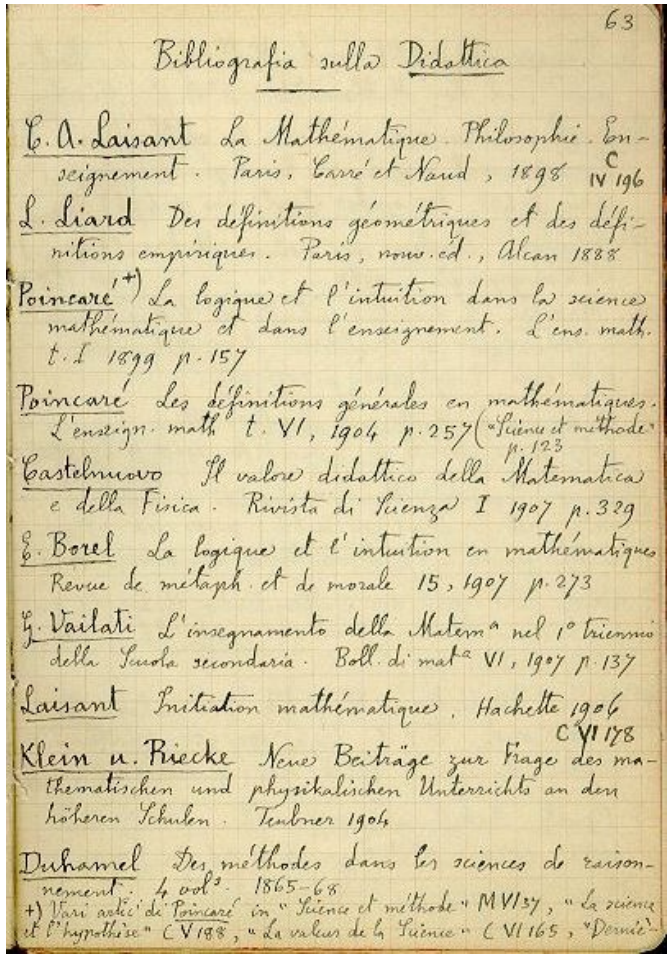
- It is important to bridge the gap between secondary and university teaching by introducing, beginning in secondary schools, the **concepts of function transformation**;
- The "**genetic" teaching method** is preferred;
- The "***first approach to mathematics should be experimental and intuitive,*** *so that the student learns "****not only to demonstrate truths*** *already known, but to* ***make discoveries****".* (p. 16)
- Applications of mathematics to other sciences (physics, astronomy, political economy, geography, etc.) make the subject more interesting and stimulating.
- **Historical perspective**.
- **Elementary mathematics from an advanced standpoint** should play a key role in teacher training.

(Rowe 1985, Schubring 1989, Giacardi 2010)

Segre's lessons to future teachers are the result of various factors :

- **Knowledge of the European reform** (Germany, France, England) **movements and debates** (ICMI);

- **His reading of papers and books concerning mathematics education or methodological issues** by Laisant, Borel, Hadamard, Poincaré (France) and Treutlein, Klein, Simon, Lietzmann (Germany), mathematicians who all criticized a teaching of mathematics too marked by logical rigour;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L. Liard Des définitions géométriques et des définitions empiriques. Paris, nouv. éd., Alcan 1888

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Poincaré Les définitions générales en mathématiques. L'enseign. math. t. VI, 1904 p. 257 ("Science et méthode" p. 123)

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G. Vailati L'insegnamento della Matem.ª nel 1° triennio della Scuola secondaria. Boll. di mat.ª VI, 1907 p. 137

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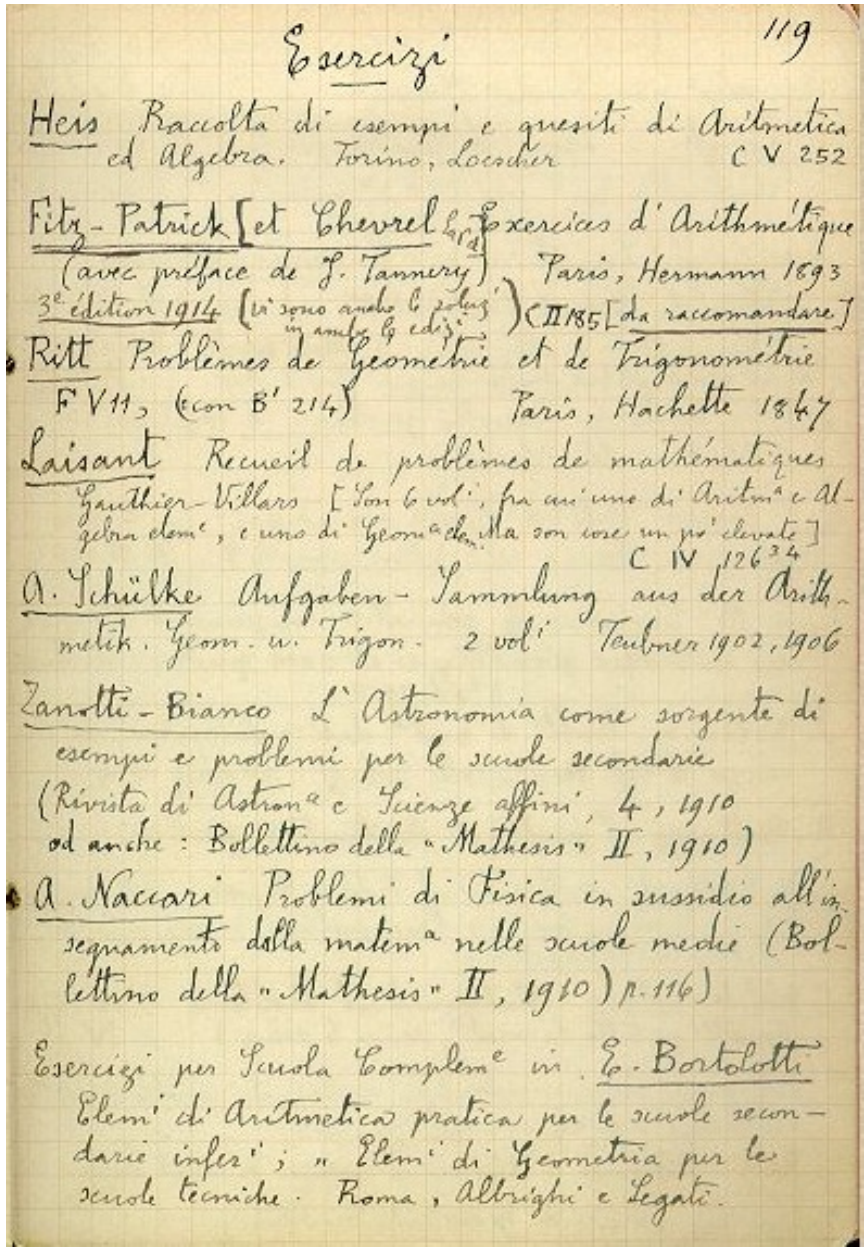
+) Vari artici di Poincaré in "Science et méthode" M VI 37, "La science et l'hypothèse" C VI 198, "La valeur de la Science" C VI 165, "Dernie"

- **The circulation of ideas from inside the School** (Castelnuovo, Enriques), but also from **outside** (Bettazzi, Vailati from Peano's School)

- Knowledge of books and articles both of **didactics in general and didactics of specific disciplines.**



Esercizi 119

Heis Raccolta di esempi e quesiti di Aritmetica ed Algebra. Torino, Loescher C V 252

Fitz-Patrick [et Chevrel] Exercices d'Arithmétique (avec préface de J. Tannery) Paris, Hermann 1893 3e édition 1914 [ci sono anche le prime 2 edizioni] C II 185 [da raccomandare]

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A. Schülke Aufgaben-Sammlung aus der Arithmetik. Geom. u. Trigon. 2 vol' Teubner 1902, 1906

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The very **rich annotated bibliography**, shows:

- Segre's **wide ranging reading** (texts dedicated to general and special didactic, foundations and history of mathematics, textbooks, scholastic programs of other European countries, etc.);
- His policies as director of the Mathematics Library of the University of Turin (1907-1924) regarding purchases of books that showed a great **attention not only to research, but also to teaching.**

(A. Conte, L. Giacardi, *Corrado Segre caposcuola e educatore,* Torino, CSSUT To appear)

Conclusion
Recognition of Segre's role as leader of a School

He has left traces of his strong mind and his great and uninterrupted efforts in a vast range of fields, … Nor should another, ***principal merit of Segre's*** *be overlooked:* ***that of having given the start to the scientific line of Italian research in Geometry on a curve and on a surface,*** *to which he himself has contributed effectively".* (*Premio reale per la Matematica,* 1898, 367)

► The feeling of belonging to a School soon appeared in Segre himself (in the 1890s) and was present within his group :

" He was truly a Maestro in the highest, most noble sense of the word… He kept up an extremely vast correspondence and stayed up to date with everything that was produced in his field of research both in Italy and abroad; he suggested problems, indicated methods, pointed out errors, always impartial in both his praise and his criticism. ***At that time, Segre, quite young, had assumed, by unanimous consensus, the role of directing the Italian school of geometry, succeeding Cremona."*** (Castelnuovo 1924, 358)

"He [Segre] became so, just in the moment in which Cremona's scientific activity had completely ceased, the new leader of Ital. geometry ***the founder of a new school"*** (Fano 1923 [*Appunti vari*], fol. 63, Fano's original English text).

"In constituting the desired School *… the most important thing of all was precisely the powerful work of Corrado Segre"* (Severi 1957, VI)

"The Italian algebraic geometry had in Corrado Segre a great Maestro and a ***new leader****"* (Terracini 1962, 11)

► Segre's role as leader and the existence of a School with a precise identity was clear not only in Italy but also abroad.

In 1923 Meyer and Mohrmann noted how in a few years at the end of the century Italy had arrived *"to a leading position"* ***(führende Stellung****)"* in the field of geometry . (EMW, 1923, III.I1, VI)

"He may probably be said to be ***the father of that wonderful Italian school*** *which has achieved so much in the birational theory of algebraical loci."* (Baker 1924, 269)

Coolidge counted Segre among the greatest architects of the *«geometric* Risorgimento *in Italy»* **(**Coolidge,1927, 352)

Conclusion
Main features of Segre's leadership

► **First of all, Segre proposed an innovative scientific project**, well defined and limited to the field of geometry, and he fostered the onset or consolidation in Italy of important lines of research;

► **He achieved a perfect symbiosis between teaching, his own personal research, and initiating young people to research.**
He directed the research of his students according to each one's personal aptitude, correcting defects and inviting them not to be "a slave to one method" in dealing with a problem:

"He [Fano] *has a good memory and a lively mind. But his tendencies are essentially geometric, for pure geometry. …* ***I believe that he would be stronger in geometry if he could be made to acquire fully the methods of analysis"*** (Segre to Klein, Torino, 4 October 1893)

► **He favoured the free circulation of ideas within the School and even outside it (Peano's School).**

► **Segre was a very demanding teacher, and rewarded excellence**:

"It is better to have one result fit to remain in science than a thousand destined to die as soon as they are born" (Segre 1891, 43).

"simple exercises' and 'useless generalisations' ***produce a 'real encumbrance in the science*** *and an embarrassment for more serious investigators'* (Segre 1904, 443).

► **He cared about assuring an academic position for his disciples**

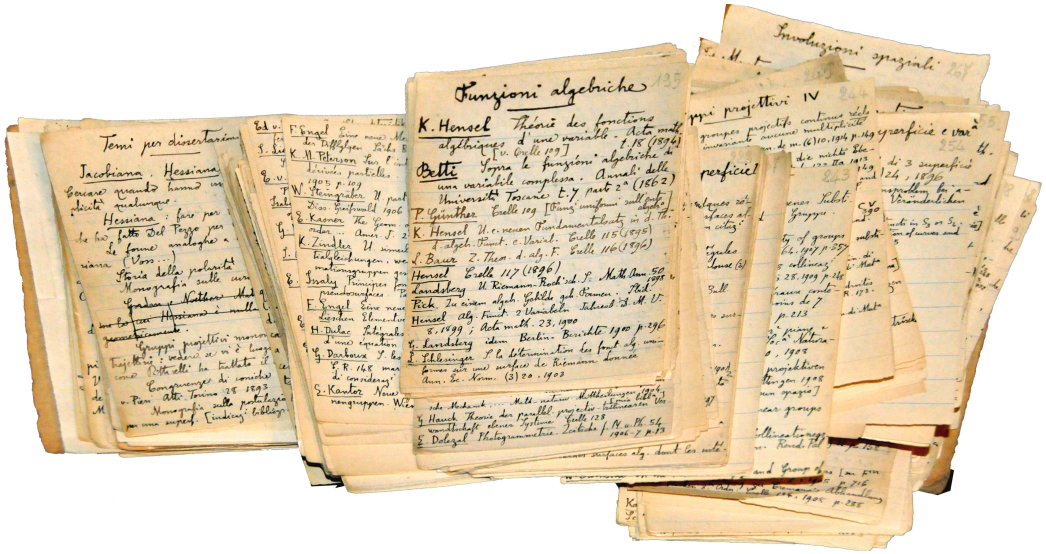
► **He took care to give the Italian tradition, and his School, national and international scientific visibility**, through personal contacts, the periods of stay abroad, the participation in international congresses, the translations and editorial activities:

- **1884-1885** - Segre wrote **35** reviews of Italian works, *"which up to then had been greatly neglected"* , for the *Jahrbuch über die Fortschritte der Mathematik*;
- **1889-1924** - he presented or reviewed as many as **189** works to be published in *Atti* or *Memorie dell'Accademia delle scienze di Torino,* in particular he saw to the **immediate publication of original results of his pupils' dissertations;**
- **1904-1924** - he was the co-director of *Annali di Matematica pura ed applicata*

► **Segre attributed great importance to the quality of university teaching**, taking particular care with the **advanced courses**, without, however, overlooking the compulsory university courses and **the training of secondary schools teachers.**

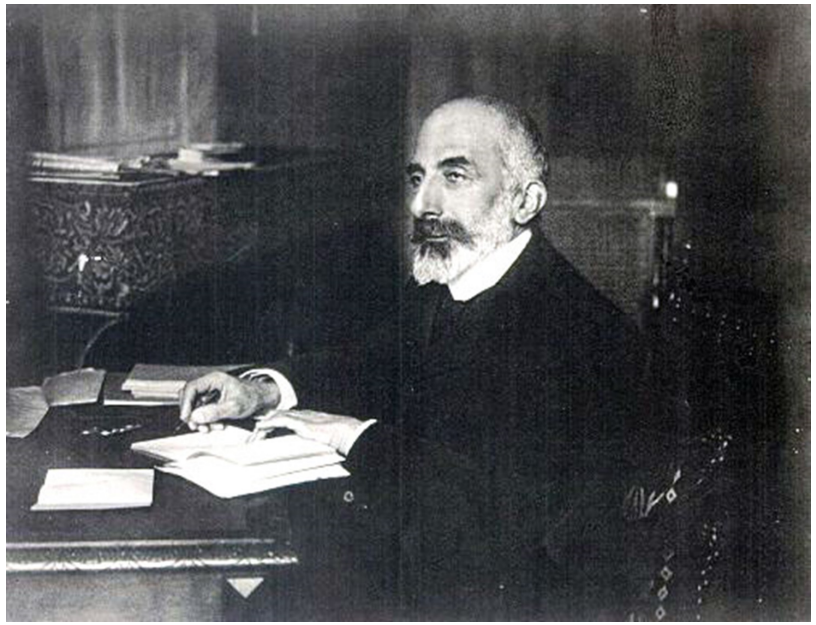
" I am very happy about what you write me regarding the impression that you have made on your students: ***this is a great victory for you: to show that your teaching skills are not inferior to your intellect.*** (C. Segre to G. Castelnuovo, 23 November 1891)

- Seminars of students (Klein's model)
- Extra hours in order to accustom students to discuss significant articles or parts of books
- Particular attention to dissertations (rich card index)



http://www.corradosegre.unito.it/doc/schedario.pdf

C. SEGRE, *Su alcuni indirizzi nelle investigazioni geometriche. Osservazioni dirette ai miei studenti* (1891; En. Transl. by J. W. Young 1904):



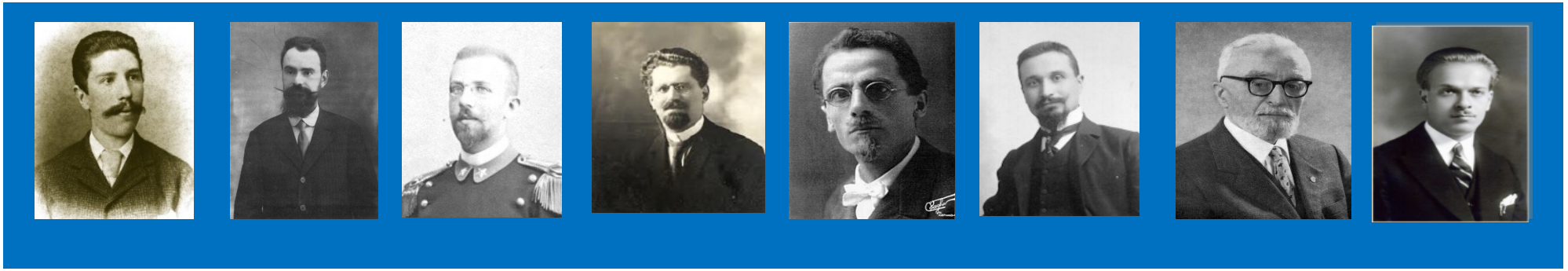
A good teacher should:

- invite his students to **deal** only **with «relevant» problems**;
- advise them to study, along with theories, their **applications**;
- urge them not to be "slave to one method", "**broaden as much is possible their own knowledge**", so as to be able to look at things "from a higher point of view";
- take due account of the **didactic needs**;
- suggest they read the works of the great masters.

A successful School

The factors that favoured the emergence of Segre's School were

- the **excellence of the students** and their important achievements;
- their **important institutional position;**
- the participation with international publishing initiatives such as the *Encyklopädie der mathematischen Wissenschaften*;
- the collaboration in international journals such as the *Mathematische Annalen*, besides the most prestigious Italian journals;
- The large network of scientific relationships



"School" in a broad sense:

- a group of researchers who from a **shared Maestro** gleaned **topics to investigate, methodologies, and approaches to research**, but who were also capable of blazing new trails independently;
- a place where **talents were developed and contacts made;**
- as an environment in which a **common vision of the transmission of knowledge matures,**
- a group with a precise national identity, but open and ready to interact with the international community, ***"a school that tends to spread itself beyond the milieu where it originated"*** (Enriques 1938, 181)

G. CASNATI, A. CONTE, L. GATTO, L. GIACARDI, M. MARCHISIO, A. VERRA, *From Classical to Modern Algebraic Geometry. Corrado Segre's Mastership and Legacy*, Springer 2016, 756 p.

Corrado Segre e la Scuola italiana di geometria algebrica, by L. GIACARDI: http://www.corradosegre.unito.it/